Abstract: The range of the Cauchy-Riemann operator for a domain $\Omega$ in a complex manifold $X$ is well understood if the complex manifold $X$ is a Stein manifold. Much less is known when the domain or the manifold is not Stein. In this talk we will discuss some recent results on the Hausdorff property of Dolbeault cohomologies. In particular, we give an example of a pseudoconvex Stein domain $\Omega$ with smooth boundary in a compact complex Hermitian manifold $X$ whose range of $\bar{\partial}$ is not closed in $L^2$.

These results are joint work with Debraj Chakrabarti and Christine Laurent-Thiébaut.